# Economics of Networks
# Introduction to Game Theory: Part 1

Evan Sadler

Massachusetts Institute of Technology

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# Agenda

Decision theory:

- Theory of rational choice
- Choice under uncertainty

Games

- Definitions
- Strategies and best responses

Pure strategy Nash Equilibrium

Examples

Suggested reading: EK chapter 6; Osborne chapters 1-3

# Motivation

Choice is a big part of economics

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How do we model this?

# Rational Choice

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Three step process of rational choice:

- What is desirable?
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Observations:

- Note desires precede recognition of feasible alternatives
- Economics says nothing about what agent should desire

# Preferences

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- For each $x, y \in X$, either $x \succeq y$ or $x \preceq y$
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Choice problem $C \subseteq X$

- Choose $\succeq$-maximal element of $C$

# Utility Representation

Associate a number $u(x)$ to each $x \in X$, *utility* of $x$

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With uncertainty, we need *cardinal* information

- How do I compare outcomes with different probabilities?

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Examples:

- An even money gamble: with probability $\frac{1}{2}$ you lose \$10, with probability $\frac{1}{2}$ you win \$10
- Getting to work: with probability $\frac{9}{10}$ it takes you 20 minutes, with probability $\frac{1}{10}$ there is road work and it takes an hour

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- Independence: if $L \prec M$, then for any $N$ and $p \in (0, 1]$, we have

$$pL + (1-p)N \prec pM + (1-p)N$$

# Choice Under Uncertainty

**Theorem**

*Suppose preferences $\preceq$ satisfy the vNM axioms. There exists a utility function $u$ on the set of outcomes $Y$ such that $L \preceq M$ if and only if*

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Expected utility theory

Suppose action $a$ induces distribution $F^a(y)$ over consequences, expected utility

$$U(a) = \int u(y) \mathrm{d}F^a(y)$$

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Notes:

- Objective versus Subjective uncertainty
- Savage (1954) axiomatizes subjective probability and subjective expected utility

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- Benefit from working hard on group work depends on others' efforts
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- How hard should I work?
- What route should I take?
- Should I tell a friend a secret?

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Do you work or shirk?

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What price do you set?

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Player order, multiple moves, and information sets captured in extensive form games

- To come later

# Normal Form Games

## Definition (Normal Form Game)

A normal form game is a triple $(N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N})$ such that

- $N = \{1, 2, ..., n\}$ is a set of players
- $S_i$ is the set of actions available to player $i$
- $u_i : S \to \mathbb{R}$ is the payoff of player $i$, where $S = \prod_{i \in N} S_i$ is the set of all action profiles

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Some notation:

- $s_{-i}$: vector of actions for all players except $i$
- $S_{-i}$: set of action profiles for all players except $i$
- $s = (s_i, s_{-i}) \in S$ is an action profile, or outcome

# Strategies versus Actions

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Distinction also exists in normal form games when we talk about mixed strategies

# The "Solution" of a Game

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Large number of "solution concepts" based on different assumptions about

- What players know about each others' plans
- How smart players are
- How smart players think others are

# Dominant Strategies

Example: The Prisoner's Dilemma

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"Confess" dominates "silence"

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**Definition**

A strategy $s_i^* \in S_i$ is *dominant* for player $i$ if for all $s_i \in S_i$ and all $s_{-i} \in S_{-i}$

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Issue: this rarely exists

# Dominated Strategies

Conversely, we might think to eliminate strategies that are dominated

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## Definition

A strategy $s_i \in S_i$ is *strictly dominated* if there exists $s_i' \in S_i$ such that for all $s_{-i} \in S_{-i}$

$$u_i(s_i', s_{-i}) > u_i(s_i, s_{-i})$$

A strategy $s_i \in S_i$ is *weakly dominated* if there exists $s_i' \in S_i$ such that for all $s_{-i} \in S_{-i}$

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No dominant strategy

- Still dominance solvable

# Iterated Deletion

More formally, define the iterative procedure:

- Step 0: Define $S_i^0 = S_i$ for each $i$
- Step k $>$ 0: Define for each $i$

$$S_i^k = \left\{ s_i \in S_i^{k-1} \,|\, \nexists\, s_i' \in S_i^{k-1} \text{ that dominates } s_i \right\}$$

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The set $S^\infty$ of strategy profiles is what survives

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**Theorem**

*Suppose that either:*

- $S_i$ *is finite for each* $i$, *or*
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*Then* $S_i^\infty$ *is nonempty for each* $i$.

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May not yield a unique prediction

# Best Responses

Another approach: suppose players make conjectures about each other

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Should play a best reply to *some* conjecture

- What conjectures should a player entertain?

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Why might this be reasonable?

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Nash Equilibrium is a standard workhorse in economic models

- Might not be reasonable in all contexts

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- Yes! Both firms earn zero profit, no way to improve.

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Similarly, $q_1^* = \frac{1-q_2-c}{2}$

# Example: Cournot Competition

What if the firms choose what quantity to produce instead?

- Both face the market price $p = 1 - q_1 - q_2$

Given $q_1$, firm $2$ chooses $q_2$ to maximize

$$q_2(1 - q_1 - q_2 - c)$$

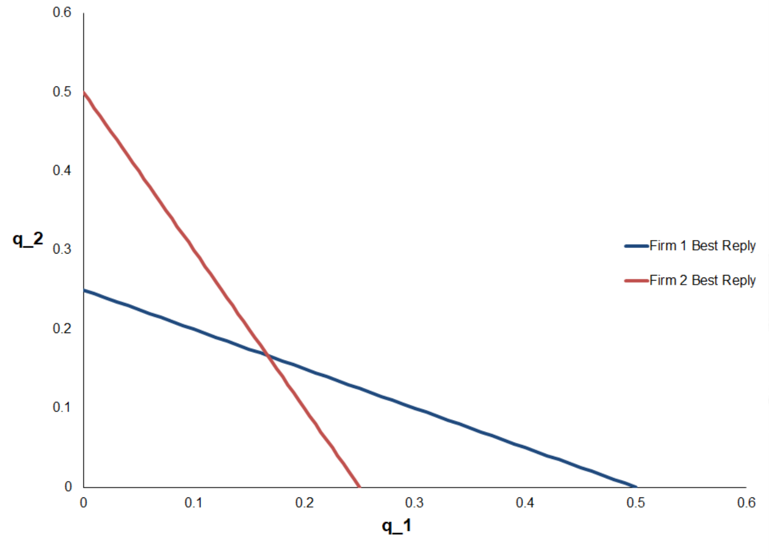
First order condition implies $q_2^* = \frac{1-q_1-c}{2}$

Similarly, $q_1^* = \frac{1-q_2-c}{2}$

Unique Nash Equilibrium:

$$q_1^* = q_2^* = \frac{2(1-c)}{3}$$

# Example: Cournot Competition



# Example: The Partnership Game

Recall our earlier choice to work or shirk:

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|---|---|---|
| Work | $(2, 2)$ | $(-1, 1)$ |
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Best reply to work is work, best reply to shirk is shirk

- Two pure strategy Nash Equilibria
- Outcome depends on conjectures

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Refinement is hard

# Example: Matching Pennies

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No pure strategy Nash Equilibrium exists

- Next time: mixed strategy equilibrium