Economics of Networks
Networked Markets

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Agenda

Perfect matchings

Bargaining

Competitive equilibrium in a two-sided market

Supply networks and aggregate volatility

Suggested Reading:

- EK chapters 10 and 11; Jackson chapter 10
- Manea (2011), "Bargaining in Stationary Networks"
- Acemoglu et al. (2012), "The Network Origins of Aggregate Fluctuations"

Buyer-Seller Networks

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Questions:

- Can every buyer (seller) find a seller (buyer)?
- Do market clearing prices exist?
- Is the outcome of trade efficient?

Perfect Matchings

A simple model:

- Disjoint sets of buyers and sellers B and S, $|B| = |S| = n$
- Bipartite graph G (all edges connect a buyer to a seller)
- Write $N(A)$ for set of neighbors of agents in A
- A *matching* is a subset of edges such that no two share an endpoint

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A matching is *perfect* if every buyer is matched to a seller and vice versa

- Contains $\frac{n}{2}$ edges

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Elegant alternating paths algorithm to find maximum matching and constricted sets (EK, 10.6)

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Consider one buyer and one seller

- Seller has an item the buyer wants
- Seller values at 0, buyer at 1
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- If reject, proceed to time 2, buyer makes offer

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Players are impatient, discount future at rate δ

The One-Shot Deviation Principle

Game has infinite time horizon, cannot use backward induction

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Theorem (Blackwell, 1965)

In an infinite horizon game with bounded per-period payoffs, a strategy profile $\mathbf{s}$ *is a SPE if and only if for each player* i *there is no profitable deviation* s_i' *that agrees with* s_i *everywhere except at a single time* t.

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HUGE simplification: only need to check deviations in a single period

- Proof is beyond our scope

Rubinstein Bargaining

Consider a profile of the following form:

- There is a pair of prices (p_s, p_b)
- The seller always proposes p_s and accepts any $p \geq p_b$
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Similarly, when buyer proposes, acceptance is incentive compatible if $p_b \geq \delta p_s$

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Theorem (Rubinstein 1982)

The alternating offers bargaining game has a unique SPE with offers (p_s^*, p_b^*) that are immediately accepted.

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Others proceed to time 2, when buyers make offers

- Alternating offers framework as before
- Previous model equivalent to a single buyer linked to a single seller

Example: Two Sellers, One Buyer

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In unique SPE, both sellers offer $p = 0$

- Logic is reminiscient of Bertrand competition
- The "short" side of the market has all the bargaining power

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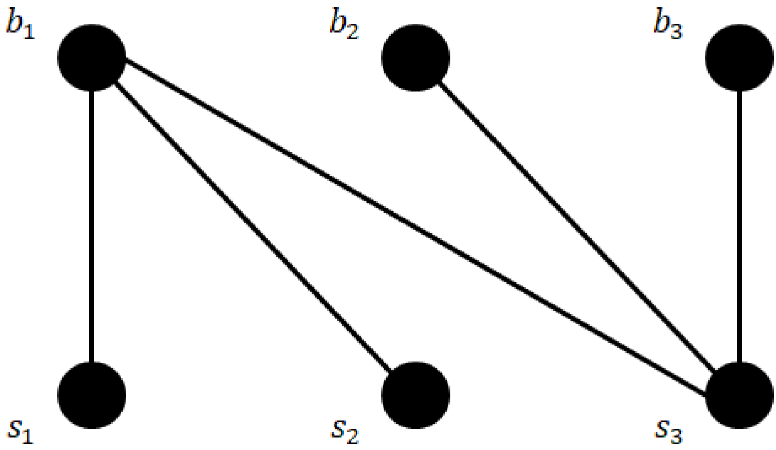
Work backwards, what happens if one pair trades and exits the market?

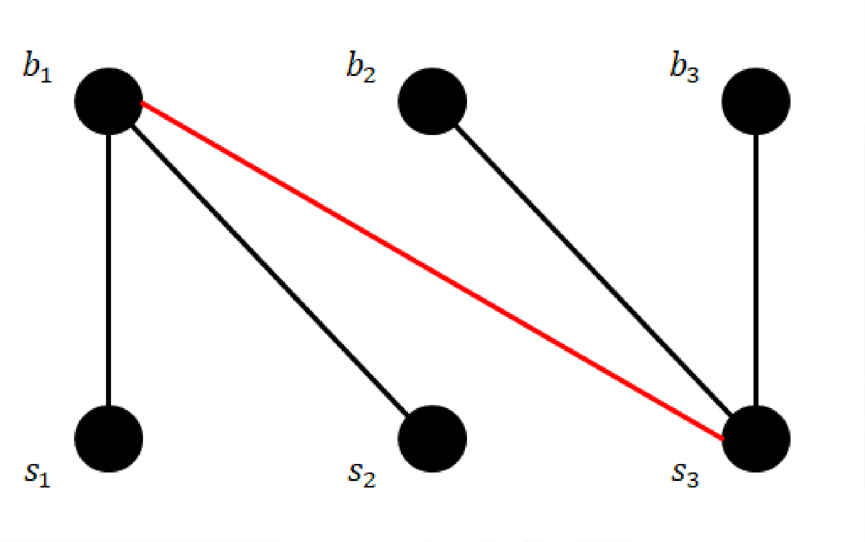
- Bargaining power is the same as in the one buyer one seller example

Bargaining in Networks

Less clear what happens in more complicated graphs



Redundant Links



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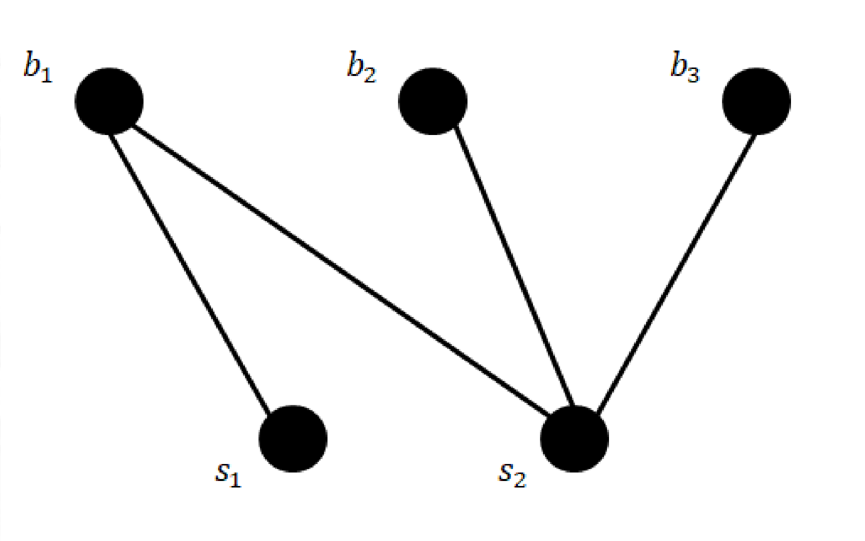
Decomposition algorithm, three sets G_S, G_B, G_E initially empty

- First, identify sets of two or more sellers linked to a single buyer, remove and add to G_S
- Next, identify remaining sets of two or more buyers linked to a single seller, remove and add to G_B
- Repeat: for each $k \geq 2$, look for sets of $k+1$ or more sellers (buyers) linked to k buyers (sellers); remove and add to the corresponding sets
- Add remaining players to G_E

Decomposition Example



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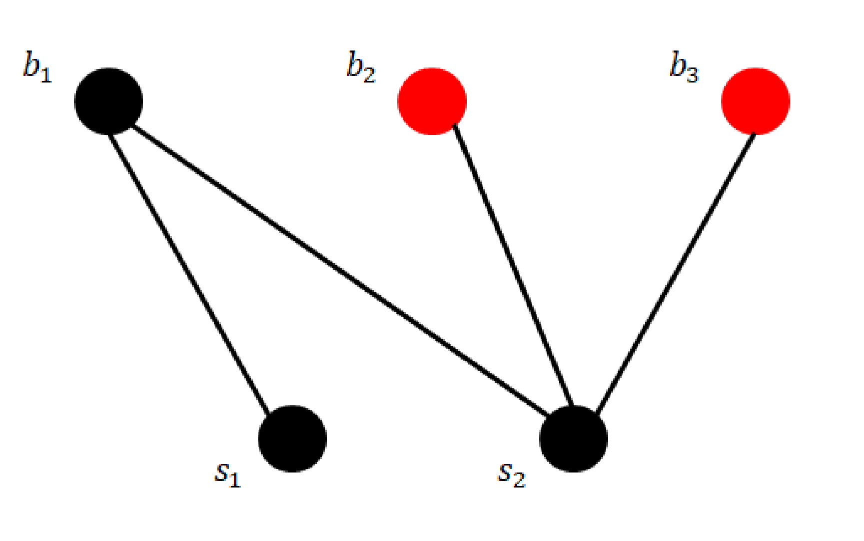

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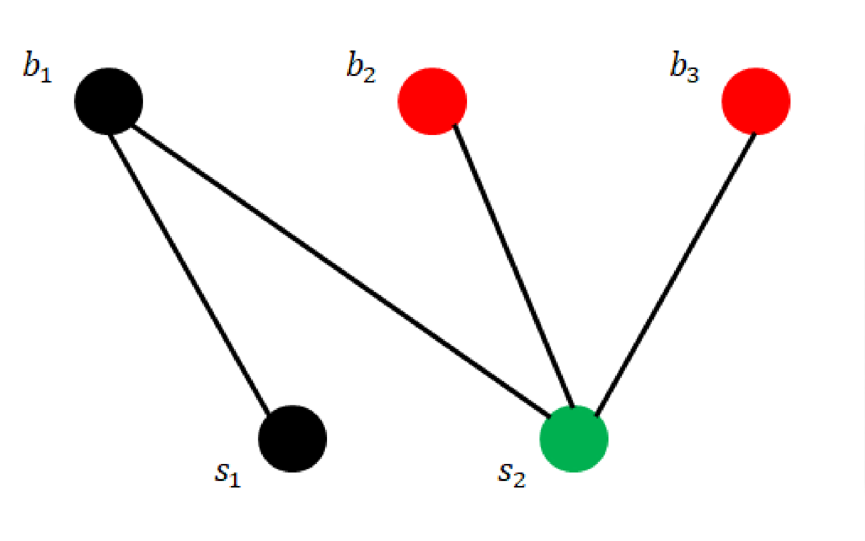

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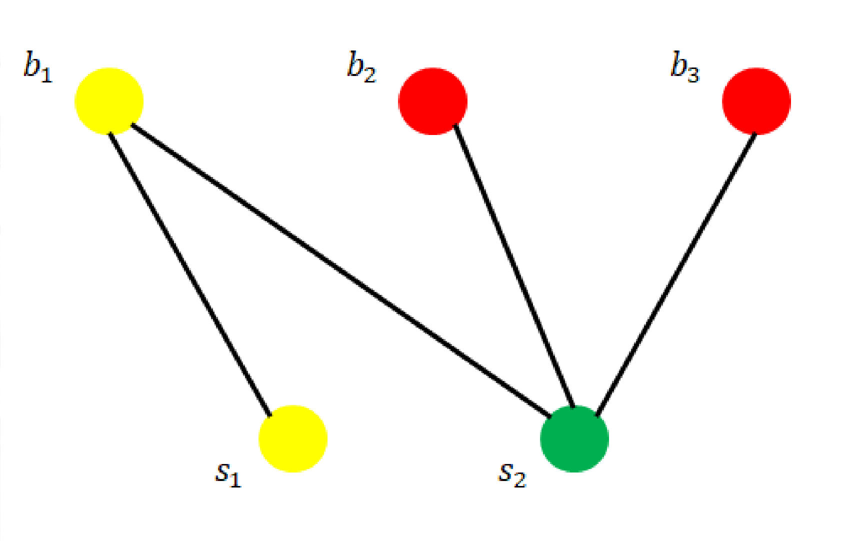

Bargaining in Networks

This simple algorithm pins down bargaining payoffs

Theorem

There exists a SPE in which:

- *Sellers in* G_S *get* 0, *buyers in* G_S *get* 1
- *Sellers in* G_B *get* 1, *sellers in* G_B *get* 0
- *Sellers in* G_E *get* $\frac{1}{1+\delta}$, *buyers in* G_E *get* $\frac{\delta}{1+\delta}$

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Prediction matches well with experimental findings

Valuations and Prices

Suppose now that buyers have heterogeneous values for different sellers' products

- Each seller has an item, values it at zero, wants to maximize profits
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Buyer i values seller j at v_{ij}, wants at most one object

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The transaction generates surplus v_{ij}

Valuations and Prices

For a buyer i, set of *preferred sellers* given prevailing prices $\mathbf{p}$

$$D_i(\mathbf{p}) = \{j \,:\, v_{ij} - p_j = \max_k [v_{ik} - p_k]\}$$

Preferred seller graph contains edge ij if and only if $j \in D_i(\mathbf{p})$

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Preferred seller graph contains edge ij if and only if $j \in D_i(\mathbf{p})$

A perfect matching in the preferred seller graph means we can match every buyer to a preferred seller, and no item is allocated to more than one buyer

- Note, whether such a matching exists will depend on the prices

Valuations and Prices

Who sells to whom?

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Definition

A price vector $\mathbf{p}$ is *competitive* if there is an assignment $\mu : B \to S \cup \{\emptyset\}$ such that $\mu(i) \in D_i\mathbf{p}$, and if $\mu(i) = \mu(i')$ for some $i \neq i'$, then $\mu(i) = \emptyset$ (i.e. buyer i is unmatched). The pair $(\mathbf{p}, \mu)$ is a *competitive equilibrium* if $\mathbf{p}$ is competitive, and additionally if seller j is unmatched in μ, then $p_j = 0$.

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Competitive equilibrium prices are market-clearing prices

- Equate supply and demand
- Corresponds to perfect matching in preferred seller graph

Existence and Efficiency

Theorem (Shapley and Shubik, 1972)

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Proof beyond our scope

More general versions of this result are known as the First Fundamental Theorem of Welfare

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Simplest stationary model:

- Set of players $N = \{1, 2, ..., n\}$
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- Common discount rate δ
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- Player i proposes a division to player j
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If accept, players exit the game and are replaced by new, identical players

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Theorem

There exists a unique payoff vector $\mathbf{v}$ *such that in every subgame perfect equilibrium, the expected payoff to player* i *in any subgame is* v_i*. Whenever* i *is selected to make an offer to* j*, we have*

- *If* $\delta(v_i + v_j) < 1$*, then* i *offers* δv_j *to* j*, and* j *accepts*
- *If* $\delta(v_i + v_j) > 1$*, then* i *makes an offer that* j *rejects*

Proof is beyond our scope; for generic δ, always have $\delta(v_i + v_j) \neq 1$

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Intuition for strategies: $\delta(v_i + v_j)$ is the joint outside option

- Players make a deal if doing so is better than the outside option for both

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Theorem

For any independent set M, we have

$$\min_{i \in M} v_i^* \leq \frac{|L(M)|}{|M| + |L(M)|}, \qquad \max_{j \in L(M)} v_j^* \geq \frac{|M|}{|M| + |L(M)|}$$

Manea (2011) provides an algorithm to compute the payoffs

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Structure of supply networks can help tell us when cascade effects are possible and how severe they might be

Supply Networks: A Model

Variant of a multisector input-output model

- Representative household endowed with one unit of labor
- Household has Cobb-Douglas preferences over n goods:

$$u(c_1, c_2, ..., c_n) = A \prod_{i=1}^{n} (c_i)^{1/n}$$

- Each good i produced by a competitive sector, can be consumed or used as input to other sectors
- Output of sector i is

$$x_i = z_i^{\alpha} l_i^{\alpha} \prod_{j=1}^{n} x_{ij}^{(1-\alpha) w_{ij}}$$

- l_i is the labor input, x_{ij} is the amount of commodity j used to produce commodity i, w_{ij} is the input share of commodity j, z_i is a sector productivity shock (independent across sectors)

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Economy characterized by a set of sectors N, distribution of sector shocks $\{F_i\}_{i \in N}$, network W

Equilibrium Output

Acemoglu et al (2012) show that the output in equilibrium (i.e. when the representative consumer maximizes utility and firms maximize profits) is given by

$$y \equiv \log(GDP) = \sum_{i=1}^{n} v_i \epsilon_i$$

where $\mathbf{v}$ is the *influence vector*

$$\mathbf{v} = \frac{\alpha}{n} \left[I - (1-\alpha) W' \right]^{-1} \mathbf{1}$$

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Shocks to more central sectors have a larger impact on aggregate output

Aggregate Volatility

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Volatility scales with the Euclidean norm of the influence vector

Example

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The influence vector then has $v_i = \frac{c}{n}$ for some c and all i

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Aggregate volatility is then

$$\sqrt{var(y)} = \sigma \sqrt{\sum_{i=1}^{n} v_i^2} = \frac{\sigma c}{\sqrt{n}}$$

Goes to zero as number of sectors becomes large

Example

Suppose we have a dominant sector 1 that is the only supplier to all others

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This implies a lower bound on aggregate volatility

$$\sqrt{var(y)} \geq \sigma_1 c$$

Volatility does not shrink with n

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Can interpret economy with large n as more disaggregated

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Theorem

Consider a sequence of economies with increasing n. Aggregate volatility satisfies

$$\sqrt{var(y)} \geq c\frac{1 + CV(d^{(n)})}{\sqrt{n}}$$

for some c.