Economics of Networks
Incomplete Information and Introduction to Social Learning

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Agenda

- Games with incomplete information
- Bayes-Nash Equilibrium
- Extensive form games
- Perfect Bayesian Equilibrium
- Rational Herding

Reading: Osborne Chapter 9; EK Chapter 16

Incomplete Information

Strategic situations often involve uncertainty

- Uncertainty about others' preferences
- Uncertainty about others' availabe strategies
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Possibility of learning affects incentives

Examples:

- Bargaining (how much is opponent willing to pay?)
- Auctions (how much do others value the object?)
- Market competition (what costs do my competitors face?)
- Social learning (What can I infer from others' choices?)

An Example

You (player 1) and a friend are trying to coordinate a meeting place (say, the mall or the library)

- Different preferences over the two options

	M	L
M	$(2, 1)$	$(0, 0)$
L	$(0, 0)$	$(1, 2)$

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Two pure strategy equilibria, one mixed

Now, suppose you are unsure whether your friend wants to meet or avoid you

Example, continued

To model this, we assume your friend (player 2) has one of two possible *types*

- One type wants to meet, the other wants to avoid

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Your friend knows which game is played, but you do not

- What are the strategies?

Example, continued

Idea: Use Nash Equilibrium in an expanded game

- Think of each type of player 2 as a separate player

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- Player 1 goes to the mall
- Player 2 goes to the mall in state 1 and the library in state 2

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Clearly a best response for player 2

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Now check for player 1

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Since both states are equally likey,

$$\mathbb{E}[u_1(M,(M,L))] = \frac{1}{2}\cdot 2 + \frac{1}{2}\cdot 0 = 1$$

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Hence, the profile $(M,(M,L))$ is a (Bayes) Nash Equilibrium

Example, continued

Note meeting at the library, player 2's preferred outcome, is no longer part of an equilibrium

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A similar calculation shows player 1 earns $\frac{1}{2}$ by playing L, but earns 1 by deviating to M

The profile $(L, (L, M))$ is not a Bayes Nash Equilibrium

Bayesian Games

More formally...

Definition

A Bayesian game consists of

- A set of players N
- A set of actions (pure strategies) S_i for each player i
- A set of types Θ_i for each player i
- A payoff function $u_i(\mathrm{s}, \boldsymbol{\theta})$ for each player i
- A (joint) probability distribution $p(\theta_1, \theta_2, ..., \theta_n)$ over types

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Note payoffs depend on vector of actions *and* vector of types

Bayesian Games, continued

We maintain the assumption that the probability distribution p is common knowledge

- Players agree on the prior probability of different type vectors

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Definition

A pure strategy for player i is a map $s_i : \Theta_i \to S_i$ prescribing an action for each type of player i.

Bayes' Rule

Recall types are drawn from the prior distribution $p(\theta_1, \theta_2, ..., \theta_n)$

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Player i evaluates expected payoff according to the conditional distribution:

$$U_i(s_i', s_{-i}, \theta_i) = \int_{\Theta_{-i}} u_i(s_i', s_{-i}(\theta_{-i}), \theta_i, \theta_{-i}) \mathrm{d}p(\theta_{-i} \mid \theta_i)$$

Bayes' Rule, continued

Quick review, basic definitions, events A and B:

- Probability of events $\mathbb{P}(A)$ and $\mathbb{P}(B)$
- Conditional probabilities $\mathbb{P}(A \mid B)$ and $\mathbb{P}(B \mid A)$
- Joint probability $\mathbb{P}(A \cap B)$

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If events are independent, then $\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$ and $\mathbb{P}(A \mid B) = \mathbb{P}(A)$

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Can also express conditional probabilities in terms of one another

- Write A^c for the complement of A
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More generally, for any countable partition $\{A_i\}_{i=1}^n$, we have

$$\mathbb{P}(A_i \mid B) = \frac{\mathbb{P}(A_i) \cdot \mathbb{P}(B \mid A_i)}{\sum_{j=1}^n \mathbb{P}(A_j) \cdot \mathbb{P}(B \mid A_j)}$$

Bayes Nash Equilibrium

Definition (Bayes Nash Equilibrium)

The profile σ is a pure strategy Bayes Nash Equilibrium if for all $i \in N$ and all $\theta_i \in \Theta_i$, we have

$$\sigma_i(\theta_i) \in \arg\max_{s_i' \in S_i} \int_{\Theta_{-i}} u_i(s_i', s_{-i}(\theta_{-i}), \theta_i, \theta_{-i}) \mathrm{d}p(\theta_{-i} \mid \theta_i)$$

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Bayes Nash equilibrium is a Nash equilibrum of the expanded game in which player i's pure strategies are maps from Θ_i to S_i

Existence of Bayes-Nash Equilibrium

Theorem

In any finite Bayesian game, a mixed strategy Bayes Nash equilibrium exists

Theorem

Consider a Bayesian game with continuous strategy spaces and types. If the strategy and type sets are compact, and payoff functions are continuous and concave in own strategies, then a pure strategy Bayes Nash equilibrium exists.

Proofs based on Kakutani's fixed point theorem are essentially identical to what we did a few weeks ago.

Example: Incomplete Information Cournot

Two firms produce an identical good at constant marginal cost

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- Cost c_l with probability θ and c_h with probability $1-\theta$, $c_l < c_h$

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- Cost c_l with probability θ and c_h with probability $1 - \theta$, $c_l < c_h$

Game has two players and two states (l and h), actions $q_i \in [0, \infty)$

Example: Incomplete Information Cournot

Payoffs for the two players are

$$u_1(q_1, q_2, t) = q_1(P(q_1 + q_2) - c)$$

$$u_2(q_1, q_2, t) = q_2(P(q_1 + q_2) - c_t),$$

where $t \in \{l, h\}$ is firm 2's type

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Three best response functions

$$B_1(q_l, q_h) = \arg\max_{q_1 \geq 0} q_1 \left(\theta P(q_1 + q_l) + (1 - \theta) P(q_1 + q_l) - c\right)$$

$$B_l(q_1) = \arg\max_{q_l \geq 0} q_l \left(P(q_1 + q_l) - c_l\right)$$

$$B_h(q_1) = \arg\max_{q_h \geq 0} q_h \left(P(q_1 + q_h) - c_h\right)$$

Example: Incomplete Information Cournot

Bayes Nash equilibria are triples (q_1^*, q_l^*, q_h^*) such that

$$B_1(q_l^*, q_h^*) = q_1^*, \quad B_l(q_1^*) - q_l^*, \quad B_h(q_1^*) = q_h^*$$

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If we take $P(Q) = \alpha - Q$, then the solution is

$$q_1^* = \frac{1}{3}\left(\alpha - 2c + \theta c_l + (1-\theta)c_h\right)$$

$$q_l^* = \frac{1}{3}\left(\alpha - 2c_l + c\right) - \frac{1-\theta}{6}(c_h - c_l)$$

$$q_h^* = \frac{1}{3}\left(\alpha - 2c_h + c\right) + \frac{\theta}{6}(c_h - c_l)$$

Note $q_l^* > q_h^*$, type with lower cost produces more

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In the game with complete information, the Nash equilibrium involves players producing

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With incomplete information, firm 2 produces more than this when its cost is c_h and less when its cost is c_l

This is because firm 1 produces a moderated output

- When firm 2 has cost c_h, firm 1 produces less than it would if it knew c_h, so firm 2 gets to produce a bit more
- When firm 2 has cost c_l, firm 1 produces more than it would if it knew c_l, so firm 2 gets to produce a bit less

Dynamic Games with Incomplete Information

We often need to think about information in dynamic settings in which players learn about the environment over time

- Look at extensive form games, explicit order of moves

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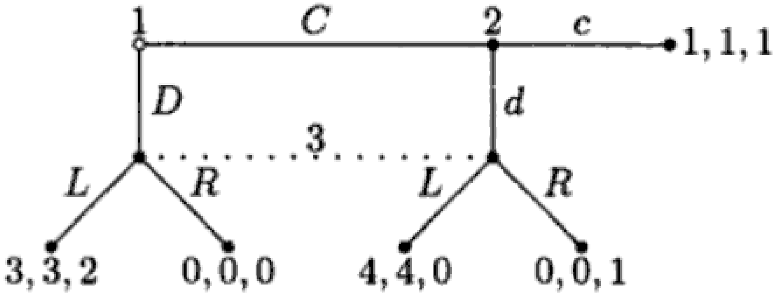
Will also refine away non-credible threats, as in subgame perfect equilibria

- New solution concept: Perfect Bayesian Equilibrium

Example

Selten's Horse:



Dynamic Games with Incomplete Information

Definition

A dynamic game of incomplete information consists of

- A set of players N
- A sequence of histories $\{h^t\}$, each assigned to a player or nature
- An information partition (which histories are in an information set)
- A set of pure strategies S_i for each player i (must include action for each information set)
- A set of types Θ_i for each player i
- A payoff function $u_i(\mathrm{s}, \boldsymbol{\theta})$ for each player i
- A joint probability distribution $p(\theta_1, ..., \theta_n)$ over types

Strategies and Beliefs

A belief system μ gives a probability distribution over nodes in each information set

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A belief system is *consistent* if it is derived from equilibrium strategies using Bayes' rule

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If player 1 chooses D with probability p and player 2 chooses d with probability q, then Bayes' rule implies

$$\mu_3(left) = \frac{p}{p + (1-p)q}$$

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What if $p = q = 0$? The consistency requirement has no bite, any belief is valid

- If an information set is not reached on the equilibrum path, beliefs are unrestricted

Perfect Bayesian Equilibrium

Definition

In a dynamic game of incomplete information, a perfect Bayesian equilibrium is a strategy profile σ and a belief system μ such that

- The profile σ is sequentially rational given μ
- The belief system μ is consistent given σ

Relatively weak solution concept, often refined by restricting off-path beliefs

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Theorem

In any finite dynamic game of incomplete information, a (possibly mixed) perfect Bayesian equilibrium exists.

Social Learning

An important set of questions in network economics:

- How much do people learn from social connections?
- Can interactions aggregate dispersed information?
- How does network structure influence what people learn?

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- People base choices on information
- We might infer information from these choices

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A simple Bayesian model gives rise to rational herding

Observational Learning

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What can we infer about the deli?

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Signals i.i.d. conditional on the state:

$$\mathbb{P}(s_n = 0 \,|\, \theta = 0) = \mathbb{P}(s_n = 1 \,|\, \theta = 1) = g > \frac{1}{2}$$

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We end up in a herd

Some Observations

Action produces an informational externality

- Agents do not internalize the value of the information they provide to others

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Herding is not sensitive to the signal structure

- Define public belief $q_n = \mathbb{P}_\sigma(\theta = 1 \mid x_1, x_2, ..., x_{n-1})$
- The public belief is a martingale and must converge

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We require some basic results on martingales

- A martingale is a stochastic process $X_1, X_2, \ldots X_t, \ldots$ such that $\mathbb{E}[X_t \,|\, X_1, \ldots, X_{t-1}] = X_{t-1}$
- Belief processes are always martingales

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Theorem (Martingale Convergence Theorem)

Suppose $\{X_t\}_{t\in\mathbb{N}}$ is a real-valued martingale and there exists a, b such that $a \leq X_t \leq b$ for all t. Then $\lim_{t\to\infty} X_t$ exists almost surely.

Proof of Theorem

Recall the public belief process $q_n = \mathbb{P}_\sigma(\theta = 1 \,|\, x_1, x_2, ..., x_{n-1})$

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- This is a martingale

By the martingale convergence theorem, $\lim_{n\to\infty} q_n$ exists almost surely

If player n chooses 1 (0), then $q_n \geq (\leq)\frac{1}{2}$

- If there are infinitely many switches, only possible limit point is $\frac{1}{2}$

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Recall the public belief process $q_n = \mathbb{P}_\sigma(\theta = 1 \mid x_1, x_2, ..., x_{n-1})$

- This is a martingale

By the martingale convergence theorem, $\lim_{n\to\infty} q_n$ exists almost surely

If player n chooses 1 (0), then $q_n \geq (\leq)\frac{1}{2}$

- If there are infinitely many switches, only possible limit point is $\frac{1}{2}$

If q_{n-1} is close to $\frac{1}{2}$, then n will follow private signal

- This implies q_n will reflect a new signal
- Inconsistent with convergence to $\frac{1}{2}$

Next Time

Next time we will enrich the model

- Instead of observing entire history, player n observes some *neighborhood* $B(n)$
- Generalize the signal structure

Will also look at a non-Bayesian approach to learning and opinion dynamics

- Based on belief-averaging procedure from DeGroot (1974)