Economics of Networks
Diffusion Part 2

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Agenda

- Recap of last time, contagion and mean-field diffusion
- The configuration model
- Diffusion in random graphs
- Monopoly pricing with word-of-mouth communication

Material not well-covered in one place. Some suggested reading: Jackson Chapter 7.2; "Word-of-Mouth Communication and Percolation in Social Networks," A. Campbell; "Diffusion Games," E. Sadler

Binary Coordination with Local Interactions

Recall our simple game:

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Cohesion can block contagion, but neighborhoods can't grow too fast

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An alternative framework

- Distributional knowledge of the network structure
- Adopt behavior iff expected payoff exceeds cost

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Key phenomenon: tipping points

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Relate steady state to network degree distribution

Random Graphs

Today, a third approach

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Stochastic outcomes, viral cascades

A Few Examples

Spread of new products through referral programs

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Spread of microfinance participation (Banerjee et al., 2013)

Questions

Basic:

- How many people adopt?
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- Who ends up adopting?

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Some implications:

- Targeted seeding
- Pricing strategies

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Look at limits as $n \to \infty$, large networks

- Assume $\{\mathbf{d}^{(n)}\}$ converges in distribution and expectation to D

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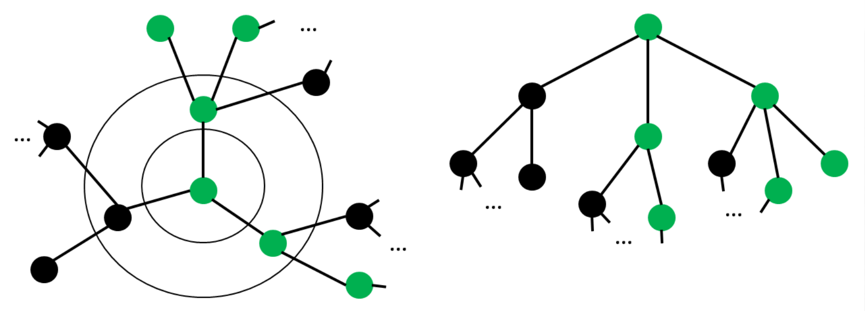
Key idea: branching process approximation

Branching Process Approximation



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And so on...

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Well-defined for $s \in [0, 1]$

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Extinction probability is unique minimal solution to $\xi = g(\xi)$

- Survival probability $\phi = 1 - \xi$

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By the martingale convergence theorem:

- As $t \to \infty$, Y_t converges almost surely
- Implication: $\phi > 0$ iff $\mu > 1$ (one exception: $Z = 1$ w.p.1)

Connecting to the Random Graph

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Subsequent nodes realize offspring according to D'

$$\mathbb{P}(D' = d) = \frac{\mathbb{P}(D = d+1) \cdot (d+1)}{\mathbb{E}[D]}$$

The Law of Large Networks

Define $\rho_k = \mathbb{P}(|\mathcal{T}| = k)$, $N_k(G)$ the number of nodes in components of size k, $L_i(G)$ the ith largest component

Theorem

Suppose $\mathbf{d}^{(n)} \to D$ *in distribution and expectation, and* $G^{(n)}$ *is generated from the configuration model with degree sequence* $\mathbf{d}^{(n)}$. *For any* $\epsilon > 0$, *we have*

$$\lim_{n\to\infty} \mathbb{P}\left(\left|\frac{N_k(G^{(n)})}{n} - \rho_k\right| > \epsilon\right) = 0, \quad \forall\, k$$

$$\lim_{n\to\infty} \mathbb{P}\left(\left|\frac{L_1(G^{(n)})}{n} - \rho_\infty\right| > \epsilon\right) = 0$$

$$\lim_{n\to\infty} \mathbb{P}\left(\frac{L_2(G^{(n)})}{n} > \epsilon\right) = 0.$$

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Proof is beyond our scope

Survival Probability of $\mathcal{T}$

Fun fact: if $g(s)$ is the generating function for D, then $\frac{g'(s)}{\mu}$ is the generating function for D':

$$g'(s) = \frac{\mathrm{d}}{\mathrm{d}s} g(s) = \frac{\mathrm{d}}{\mathrm{d}s} \sum_{k=0}^{\infty} \mathbb{P}(D = k) s^k$$

$$= \sum_{k=1}^{\infty} k \mathbb{P}(D = k) s^{k-1}$$

$$= \sum_{k=0}^{\infty} (k+1) \mathbb{P}(D = k+1) s^k$$

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If ξ solves $\mu \xi = g'(\xi)$, survival probability of $\mathcal{T}$ is $\phi = 1 - g(\xi)$

- Giant component covers fraction ϕ of the network

Typical Distances

Define $\nu = \mathbb{E}[D']$, $H(G)$ distance between two random nodes in the largest component of G

Theorem

A giant component exists if and only if $\nu > 1$. In this case, for any $\epsilon > 0$ we have

$$\lim_{n\to\infty} \mathbb{P}\left(\left|\frac{H(G)}{\log_\nu n} - 1\right| > \epsilon\right) = 0$$

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Typical distance between nodes is $\log_\nu n$

- Relates to growth rate of the branching process $\mathcal{T}$

A Diffusion Process

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Suppose n individuals are linked in a configuration model, and one random person starts out with the product

- How many people end up buying?
- How long does it take to spread?

Outcome Variables

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Will characterize α_n and τ_n for large n

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Generating function for D_p:

$$g_p(s) = g\left(p + (1 - p)s\right)$$

The Extent of Diffusion

Recall $\mu = \mathbb{E}[D]$

Theorem

There exist ϕ_p and ζ_p such that α_n converges in distribution to a random variable α, taking the value ϕ_p with probability ζ_p and the value 0 otherwise. To obtain these constants, we can solve

$$\mu\xi = g'\left(p + (1-p)\xi\right)$$

If ξ^* is the solution, we have $\phi_p = (1-p)\left(1 - g_p(\xi^*)\right)$ and $\zeta_p = 1 - g_p(\xi^*)$.

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Intuitively, question is whether the initial seed touches the giant component in the percolated graph

- ϕ_p is fraction of nodes in the component, ζ_p is fraction of nodes that link to this component

The Extent of Diffusion

$\mathbb{E}[D_p] = (1-p)\mu$, $g_p'(s) = (1-p)g'(p + (1-p)s)$

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Probability that I do not link to the giant component:

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Probability that I purchase: $1-p$

The Rate of Diffusion

Theorem

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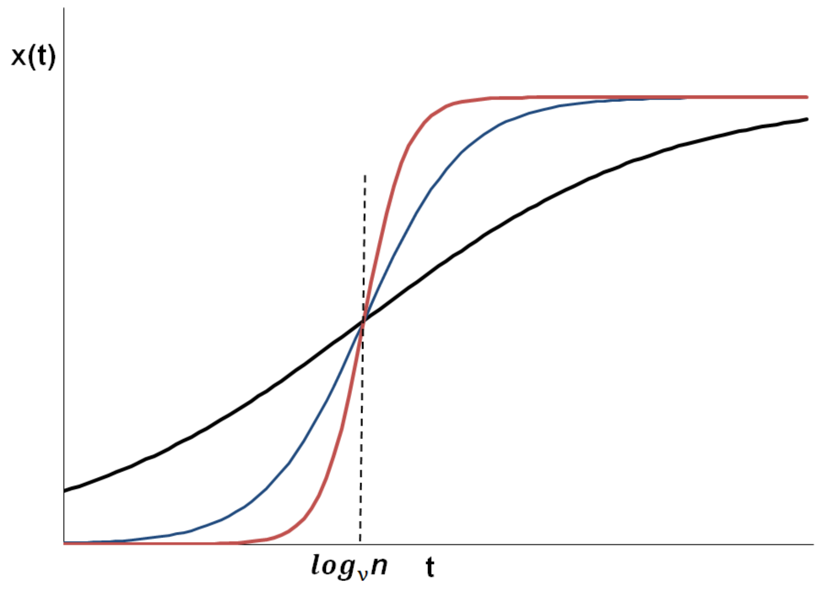
The time it takes to reach any positive fraction is roughly $\log_{p\nu} n$

The Rate of Diffusion



Comparative Statics

How does adoption change with the price and the network?

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Theorem

Suppose $\nu > 1$. *There exists a critical price* $p_c \in (0, 1)$ *such that*

- $\phi_p = \zeta_p = 0$ *for* $p \geq p_c$
- $\phi_p > 0$ *for* $p < p_c$, *and* $\frac{\partial \phi_p}{\partial p} < 0$

Suppose D *and* $\hat{D}$ *are two distributions, with*ϕ_p *and* $\hat{\phi}_p$ *the corresponding giant component sizes, and* ν *and* $\hat{\nu}$ *the corresponding forward degrees. If* D *FOSD* $\hat{D}$, *then* $\phi_p \geq \hat{\phi}_p$ *and* $\nu \geq \hat{\nu}$. *If* D *is a mean preserving spread of* $\hat{D}$, *then* $\nu \geq \hat{\nu}$

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Making the network more dense leads to more adoption and faster diffusion

- Mean preserving spread makes diffusion faster, but may not lead to more adoption

Example

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Under $\hat{D}$, extinction probability solves

$$6\xi = 1 + 5\left(p + (1-p)\xi\right)^4$$

For p close to zero, ξ close to 0.17, $\phi_p \approx 0.83$

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Choose p to maximize $Q(p)(p - c)$

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If all consumers were exposed to the product, then $Q(p) = 1 - p$

- Maxmizie $(1-p)(p-c)$
- Set $p = \frac{1+c}{2}$, profit $\frac{(1-c)^2}{4}$
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With word of mouth, $Q(p) = (1 - p)\,(1 - g_p(\xi^*))$, strictly less

- Demand is also more elastic:

$$\frac{p}{Q(p)} \frac{\partial Q}{\partial p} = -\frac{p}{1 - p}\left(1 + \frac{(1 - p)(1 - \xi^*)g'\,(p + (1 - p)\xi^*)}{1 - g_p(\xi^*)}\right)$$

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Implies lower optimal price

Price Comparative Statics

Recall Poisson distribution:

$$\mathbb{P}(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Mean and variance λ

Theorem (Campbell, 2013)

Suppose the degree distribution is Poisson with parameter λ. The optimal monopoly price is increasing in λ.

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Dense network leads to higher prices

- Intuition: monopolist less reliant on any individual spreading information

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New objective, maximize

$$Q(p, \omega)(p - c) - \alpha\omega$$

Quantity depends now both on price and on advertising ω

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All else equal, higher prices tend to go with more advertising

Takeaways

Discrete network diffusion models help us think about viral cascades

- Component sizes in percolation network

Faster diffusion $\neq$ more diffusion

Word-of-mouth leads to more elastic demand, tends to lower prices

Next time: models of network formation